## Approximation via Comparison: Mandarin duō in Numerical Expressions

**Data** Recently there have been discussions about approximation-inducing morphemes in numerical expressions, e.g. *thirty-some* in Anderson 2015, 2020 and Mendia 2018. The current paper joins this empirical investigation and focuses on an adjective  $du\bar{o}$  in Mandarin Chinese. It can attach to a Numeral Phrase (NumeralP) or Measure Phrase (MeasureP) to refer to a bounded interval in (1).

(1)	a.	[ <sub>NumeralP</sub> [sān shí] <b>duō</b> ] gè dàngāo	$\sim$ (30, 40)
		three ten DUO CL cake	
	b.	$[_{MeasureP} [w\check{u} m\check{i}] du\bar{o}] b\check{u}$	$\sim$ (5m, 6m)
		five cl <sub>meter</sub> Duō cloth	

 $du\bar{o}$  is different from *some* in that it is not a determiner but an adjective, so the determiner-based approach by Anderson and Mendia cannot work for  $du\bar{o}$ . (2) shows that as an ordinary adjective in Mandarin  $du\bar{o}$  is inherently comparative, i.e. able to encode comparison without overt comparative morphemes. Following the spirit in Anderson and Mendia's analyses of *some*, we are led by considerations of consistency to propose a comparison-based treatment of approximative  $du\bar{o}$ .

(2) wŏ-de shū bĭ nĭ-de shū duō
1sg-gen book than 2sg-gen book duō
'I have more books than you.'

**Analysis** We propose that approximative  $du\bar{o}$  is a degree adjective that measures and compares degrees directly, and that X- $du\bar{o}$  (X a NumeralP/MeasureP) is a comparative construction semantically. We offer a formal implementation of the idea in three steps as follows.

**Step 1: From degree to interval** It can be readily assumed that NumeralP and MeasureP are names of degrees. Considering the syntactic overlapping distribution of precise and imprecise numerical expressions, we assume that they have the same semantic type; that is, both  $s\bar{a}n shi$  and  $s\bar{a}n shi du\bar{o}$  denote intervals, i.e. convex sets of degrees, hence of type  $\langle d, t \rangle$ . For instance,  $s\bar{a}n shi$  denotes a singleton  $\{30\}$  or [30, 30] and  $s\bar{a}n shi du\bar{o}$  denotes (30, 40). Arithmetic operations like addition and multiplication can be extended to intervals by pointwise application (Moore 1979).

(3)  $I_1 \text{ op } I_2 = \{ d_1 \text{ op } d_2 \mid d_1 \in I_1 \text{ and } d_2 \in I_2 \}$ , where op stands for  $+, -, * \text{ or } \div$ .

**Step 2: Upper bound as implicature** X- $du\bar{o}$  denotes an interval that has an upper bound and lower bound intuitively. However, we claim that the upper bound is an implicature, unnecessary to calculate in the truth-conditions. Evidence comes from the reinforcement test in (4), which is also used by Anderson 2020 to show the upper bound of X-*some* has a similar nature.

- (4) Zhāngsān zŏnggòng mǎi-le sānshí-duō běn shū, ??bùzhǐ sānshí běn / bùdào Zhāngsān in.total buy-ASP thirty-DUŌ CL book more.than thirty CL less.than sìshí běn
  - forty CL

'Zhāngsān bought 30-duō books in total, more than 30 / less than 40.'

**Step 3: Comparative construction** Now we know that  $[X-du\bar{o}]$  is simply  $([X], +\infty)$  truthconditionally, similar to the comparative modified numeral *more than* X. We show that this is not an coincidence by proposing that  $du\bar{o}$  is an adjective that encodes comparison inherently. Following Zhang & Zhang-Yukun 2025, adjectives encode measure functions and take a differential and a comparison standard interval as arguments. Specifically, we propose that  $du\bar{o}$  encodes an underspecified measure function AMOUNT(·). When  $du\bar{o}$  measures entities, its entry can be formalized in (5). When  $du\bar{o}$  measures events in (6), its entry can be formalized in (7).

- (5)  $\llbracket du\bar{o}_e \rrbracket = \lambda I_{diff} \lambda I_{stdd} \lambda x. \ I_{diff} \subseteq (0, +\infty). \ \text{amount}(x) \subseteq \iota I[I I_{stdd} = I_{diff}]$
- (6) wǒ bǐ nǐ duō-pǎo-le liǎng gōnglǐ 1sg than 2sg duō-run-Asp two kilometer 'I ran two more kilometers than you.'
- (7)  $\llbracket \operatorname{du}\bar{o}_v \rrbracket = \lambda P \lambda I_{\text{diff}} \lambda I_{\text{stdd}} \lambda e. P(e) \wedge I_{\text{diff}} \subseteq (0, +\infty). \text{ amount}(e) \subseteq \iota I[I I_{\text{stdd}} = I_{\text{diff}}]$

We assume that  $AMOUNT(\cdot)$  returns degrees along dimensions like cardinalities, volume, distance, etc. restricted by monotonicity constraints (cf. Wellwood 2019). In X-*duō*, it measures degrees directly and returns a singleton of that degree as in (8).

(8)  $\llbracket du\bar{o}_d \rrbracket = \lambda I_{\text{DIFF}} \lambda I_{\text{STDD}} \lambda d. \ I_{\text{DIFF}} \subseteq (0, +\infty). \text{ Amount}(d) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ or equivalently,  $\lambda I_{\text{DIFF}} \lambda I_{\text{STDD}}. \ I_{\text{DIFF}} \subseteq (0, +\infty). \ \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ 

We analyze the derivation of  $s\bar{a}n shi du\bar{o}$  as in (9).  $du\bar{o}$  takes two arguments, one differential denoted by a covert DEGREE (encoding a default interval  $(0, +\infty)$ , cf. NUMBER in Anderson 2020), one standard denoted by  $s\bar{a}n shi$ , as the difference and subtrahend respectively, and calculates the minuend as output. Evidence for a covert DEGREE comes from overt counterparts in Chinese dialects such as cin in Fuyang Wu and *lei* in Wenzhou Wu. In this way,  $du\bar{o}$  resembles ADD, the addition operator in numerical expressions, which implies that X- $du\bar{o}$  also instantiates the additive structure, confirming the parallelism between comparison and additivity (Zhang & Zhang-Yukun 2025).

- (9) a. LF of sān shí duō:  $[NumeralP [ sān shí ]_{standard} duō DEGREE_{differential} ]$ 
  - b.  $[[NumeralP]] = [[du\bar{o}]]([[begree]])([[s\bar{a}n shi]]) = \iota I[I [30, 30] = (0, +\infty)] = (30, +\infty)$

Account for the upper bound Note that the upper bound of X- $du\bar{o}$  coincides with the syntactic upper limit of the values following and being added to X. For instance, the upper bound of  $s\bar{a}n shi$   $du\bar{o}$  is 40, and expressions like  $*s\bar{a}n shi shi$  (intended to mean 40) and larger are ungrammatical. This parallelism between  $du\bar{o}$  and ADD points to a common constraint in numerical system, which we dub as Digit Preservation Condition (DPC). DPC requires that once the tenth digit is specified as 3 in  $s\bar{a}n shi$ , it cannot be altered to 4 or larger, deriving both syntactic and semantic consequences.

(10) a.  $s\bar{a}n shi du\bar{o} \rightsquigarrow (30, 40) \Leftrightarrow *30 \text{ ADD } 10, *30 \text{ ADD } 11, \text{ etc.}$ b.  $si b\check{a}i du\bar{o} \rightsquigarrow (400, 500) \Leftrightarrow *400 \text{ ADD } 100, *400 \text{ ADD } 101, \text{ etc.}$ 

(11) **The Digit Preservation Condition (DPC)** 

The value of a digit place should be specified once and for all in the derivation of numerical expressions.

**Theoretical Implications** There are several theoretical points that deserve to be highlighted in our analysis of Mandarin  $du\bar{o}$ : (i) A phrasal-level analysis of numerical expressions is made necessary when taking into account approximation-inducing morphemes like  $du\bar{o}$  and *some*, which retain their semantic characteristics from uses outside numerical expressions. (ii) The comparative construction can be viewed as an instantiation of the additive structure. In our case the additive structure exists not across or within sentences, but even within (number) words. (iii) DPC implies that there are independent rules functioning in the derivation of numerical expressions and having their cognitive root in how human-beings carry out counting and measuring activities by unit.

## References

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